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THE ASYMMETRY EFFECTS IN A SPATIAL OLIGOPOLY MODEL

ЕФЕКТИ АСИМЕТРІЇ В МОДЕЛІ ПРОСТОРОВОЇ ОЛІГОПОЛІЇ

ANNOTATION
This paper generalizes model of the spatial duopoly [1] to analyze the effects of asymmetry. The location asymmetry of the firms and the asymmetry of the markets sizes are considered. The competition game consists of two stages. In the first stage, the firms simultaneously select their locations. In the second stage, given the location decisions, the firms simultaneously choose their supplied quantities. The equilibrium of the model is solved by backward induction. It is obtained that at a Nash equilibrium, increasing of firms numbers, markets size asymmetry and potential of markets will be conduce to agglomeration of firms on the large market. The increase in unit transportation costs will be conduce to dispersion of firms.

Keywords: spatial oligopoly, location asymmetry, markets sizes asymmetry, Nash equilibrium.

АНОТАЦІЯ
У даній роботі узагальнюється модель просторової дуополії [1] та аналізуються ефекти асиметрії. Розглянуто асиметрії розмірів ринків і розташування фірм. Конкурентна гра складається з двох етапів. На першому етапі фірми одночасно вибирають своє місце розташування. На другому етапі, враховуючи рішення про місце розташування, фірми одночасно вибирають свої обсяги пропозиції. Отримано, що в стані рівноваги Неша, збільшення числа фірм, асиметрії розмірів ринків і потенціалів ринків сприяє агломерації фірм на великому ринку. Зростання транспортних витрат сприяє дисперсії фірм.

Ключові слова: просторова олігополія, асиметрія розташування, асиметрія розмірів ринків, рівновага Неша.

АННОТАЦИЯ
В данной работе обобщается модель пространственной дуополии [1] и анализируются эффекты асимметрии. Рассмотрены асимметрии размеров рынков и местоположения фирм. Конкурентная игра состоит из двух этапов. На первом этапе фирмы одновременно выбирают свое местоположение. На втором этапе, учитывая решения о местоположении, фирмы одновременно выбирают свои объемы предложения. Получено, что в состоянии равновесия Нэша, увеличение числа фирм, асимметрии размеров рынков и потенциалов рынков способствует агломерации фирм на большом рынке. Рост транспортных расходов способствует дисперсии фирм.

Ключевые слова: пространственная олигополия, асимметрия местоположения, асимметрия размеров рынков, равновесие Нэша.

Problem setting and publications analysis. After the appearance of the famous Hotelling's work [2], problems of agglomeration and dispersion of firms in a space became a constant subject of economists study. In the case of price competition, firms will be dispersed, as with agglomeration their profits will decrease to zero due to the Bertrand paradox [3]. In the case of quantitative competition, firms will tend to agglomerate [4], [5].

Investigation of agglomeration and dispersion of firms depending on transport costs and market sizes was carried out in [1]. The paper [1] develops a barbell model [6] with homogeneous product and asymmetric demands to compare prices, aggregate profits and social welfare between Cournot and Bertrand competition, and to analyze the firms' equilibrium locations. It focuses on the impacts of the spatial barrier generated from transport costs, and the market size effect resulting from asymmetric demands. It shows that the market-size effect is crucial in determining firms' locations under Cournot competition, but insignificant under Bertrand competition.

In the paper [6] have studied the effects of spatial price discrimination on output, welfare and location of a monopolist in the context of spatial economy. It is shown that a monopoly will locate at different markets under different pricing schemes. Specifically, if the slope of the demand function in one market is higher than that in another market, then a monopoly will locate at the former market under simple mill pricing, while it will locate at the latter market under discriminatory pricing.

The paper [1] was developed in the paper [7]. The paper [7] considers a spatial discrimination Cournot model with asymmetric demand. In the model used the geographical interpretation of the linear market and introduce differentiated products. It is analyzed a location-quantity game and shown that agglomeration or dispersed locations may arise, depending on parameter combinations.

Formulation of research objectives. As well-known, one of a promising areas of the study for spatial models are the effects of asymmetry [8]-[10]. The **aim of this article** is to generalize the model [1] and analyze the asymmetry effects.

The basic results and their justification. Suppose that there are two markets, which are located at the endpoints of the line with a length of l. The markets are connected by a road or a highway. There is a size asymmetry between markets. Assume that a size of the left market (L-market) exceeds a size of the right market (S-market). There are n competing firms, which can locate at any point along a line. In both markets firms sell homogeneous goods and arbitrage among consumers is excluded. Each firm faces linear transportation costs of t to move one good unit per one unit of distance.

A distance of the i-th firm to the L-market is x_i, $i \in N$, $N = \{1, 2, ..., n\}$ – set of firms. There is a location asymmetry between firms: $x_1 \le x_2 \le ... \le x_n$ (Fig. 1).

Each firm chooses an optimal location, which can be at one of the two markets or a point on the line. The barbell model fits the reality well, and can be used to examine the trade between two countries as well.

The linear demand curves at each market: $p^L = b - \frac{k}{\gamma} \cdot \sum_{i \in N} q_i^L$, $p^S = b - k \cdot \sum_{i \in N} q_i^S$,where p^L , p^S – the market prices, q_i^L , q_i^S – the quantities supplied of i -th firm, b – the minimum price at which there is no demand (market potential), k – is a coefficient of price sensitivity, $\gamma \ge 1$ – is a size-markets asymmetry coefficient (Fig. 2).

The competition game consists of two stages. In the first stage, the firms simultaneously select their locations. In the second stage, given the location decisions, the firms simultaneously choose their supplied quantities. The equilibrium of the model is solved by backward induction.

The profit of i -th firm are defined as the sum of its profits from both markets

$$F_i = q_i^L \cdot \left(b - \frac{k}{\gamma} \cdot \sum_{i \in N} q_i^L - t \cdot x_i \right) + + q_i^S \cdot \left(b - k \cdot \sum_{i \in N} q_i^S - t \cdot (l - x_i) \right) \to \max_{x_i, q_i^L, q_i^S} . \quad (1)$$

The analysis starts with the second stage. First we find the optimal volumes of supplies. The first-order conditions are as follows:

$$\frac{\partial F_i}{\partial q_i^L} = b - \frac{2 \cdot k}{\gamma} \cdot q_i^L - \frac{k}{\gamma} \cdot \sum_{j \in N \setminus i} q_j^L - t \cdot x_i = 0 ,$$
$$\frac{\partial F_i}{\partial q_i^S} = b - 2 \cdot k \cdot q_i^S - k \cdot \sum_{j \in N \setminus i} q_j^S - t \cdot (l - x_i) = 0 , \quad (2)$$

with the second-order conditions:

$$\frac{\partial^2 F_i}{\partial \left(q_i^L\right)^2} = -\frac{2 \cdot k}{\gamma} < 0 , \quad \frac{\partial^2 F_i}{\partial \left(q_i^S\right)^2} = -2 \cdot k < 0 , \quad i \in N .$$

Solving equations system (2) yields Cournot equilibrium volumes of supplies

$$\left(q_i^L\right)^* = \frac{\gamma \cdot \left(b - n \cdot t \cdot x_i + t \cdot \sum_{j \in N \setminus i} x_j \right)}{k \cdot (n+1)} ,$$
$$\left(q_i^S\right)^* = \frac{b - t \cdot l + n \cdot t \cdot x_i - t \cdot \sum_{j \in N \setminus i} x_j}{k \cdot (n+1)} . \quad (3)$$

The equations (3) show that the optimal volumes of the i -th firm increase when approaching the market and when competitors are far from the market: $\partial \left(q_i^L\right)^* / \partial x_i < 0$, $\partial \left(q_i^L\right)^* / \partial x_j > 0$, $\partial \left(q_i^S\right)^* / \partial x_i > 0$, $\partial \left(q_i^S\right)^* / \partial x_j < 0$.

The optimal prices and profits

$$\left(p^L\right)^* = \frac{b + t \cdot \sum_{i \in N} x_i}{n+1} ,$$
$$\left(F_i^L\right)^* = \frac{\gamma \cdot \left(b - n \cdot t \cdot x_i + t \cdot \sum_{j \in N \setminus i} x_j \right)^2}{k \cdot (n+1)^2} ,$$
$$\left(p^S\right)^* = \frac{b + n \cdot t \cdot l - t \cdot \sum_{i \in N} x_i}{n+1} , \quad (4)$$
$$\left(F_i^S\right)^* = \frac{\left(b - t \cdot l + n \cdot t \cdot x_i - t \cdot \sum_{j \in N \setminus i} x_j \right)^2}{k \cdot (n+1)^2} ,$$
$$F_i^* = \left(F_i^L\right)^* + \left(F_i^S\right)^* .$$

In the first stage each firm selects a profit-maximizing location given the rival's location. Substitution of (3) into (1) and differentiation with respect to location gives

$$\frac{\partial F_i^*}{\partial x_i} = -\frac{2 \cdot \gamma \cdot n \cdot t \cdot \left(b - n \cdot t \cdot x_i + t \cdot \sum_{j \in N \setminus i} x_j \right)}{k \cdot (n+1)^2} + + \frac{2 \cdot n \cdot t \cdot \left(b - t \cdot l + n \cdot t \cdot x_i - t \cdot \sum_{j \in N \setminus i} x_j \right)}{k \cdot (n+1)^2} = 0 , \quad (5)$$

with the second-order condition:

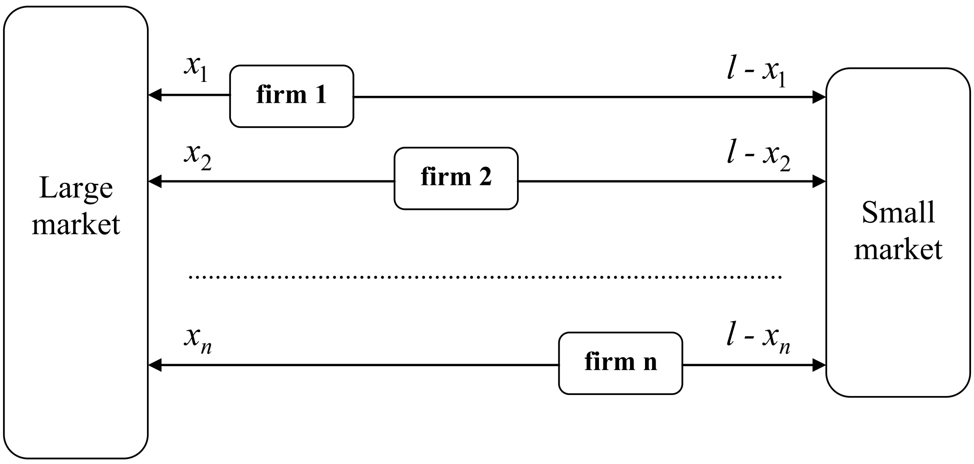

Fig. 1. The spatial oligopoly model (barbell model)

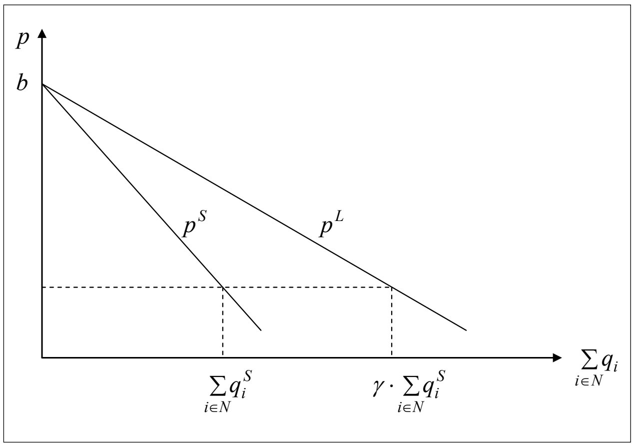

Fig. 2. The markets sizes asymmetry

$$\frac{\partial^2 F_i^*}{\partial x_i^2} = \frac{2 \cdot n^2 \cdot t^2 \cdot (\gamma + 1)}{k \cdot (n+1)^2} > 0, \ \mathrm{i} \in N. \qquad (6)$$

From the second-order condition **(6)** it follows that the profit function of i-th firm **(4)** is strictly convex with respect to x_i. Thus, at an equilibrium state firms will locate only on markets, i.e. $x_i^e = 0$ or $x_i^e = l$. We note that this result was first obtained in **[6]**.

As we know, sometimes it is useful to know the worst solution. Solving equations system **(4)** yields: $x_i^{worst} = \frac{l}{\gamma+1} + \frac{b \cdot (\gamma - 1)}{t \cdot (\gamma + 1)}$. With the growth of market sizes asymmetry, the worst solution will be to move away from the L-market: $\frac{\partial x_i^{worst}}{\partial \gamma} = \frac{2 \cdot b - t \cdot l}{t \cdot (\gamma+1)^2} > 0$.

Let us find a Nash equilibrium in this model. In conditions of location asymmetry, the equilibrium distribution of firms between markets will depend on the firm for which the choice of the market does not matter. Such an equilibrium firm is found from condition:

$$\Delta = F_i^*(x_i = 1) - F_i^*(x_i = 0) = 0,$$

$$F_i^*(x_i = 0) = \frac{\gamma \cdot (b + t \cdot l \cdot (n - i))^2}{k \cdot (n+1)^2} + \frac{(b - t \cdot l \cdot (n - i + 1))^2}{k \cdot (n+1)^2},$$

$$F_i^*(x_i = l) = \frac{\gamma \cdot (b - t \cdot l \cdot i)^2}{k \cdot (n+1)^2} + \frac{(b + t \cdot l \cdot (i - 1))^2}{k \cdot (n+1)^2},$$

$$\Delta = \frac{n \cdot t \cdot l}{k \cdot (n+1)^2} \cdot (t \cdot l \cdot ((\gamma + 1) \cdot (2 \cdot i - n) - 2) - 2 \cdot b \cdot (\gamma - 1)) = 0. \qquad (6)$$

From **(6)** we find an index of the equilibrium firm

$$i^e = \frac{n}{2} + \frac{1}{\gamma + 1} + \frac{b \cdot (\gamma - 1)}{t \cdot l \cdot (\gamma + 1)}, \ \mathrm{n}/2 < i^e \le n. \qquad (7)$$

At the Nash equilibrium on the L-market will be located firms with index $i < i^e$, on the S-market will be located firms with index $i > i^e$. It follows from **(7)** that at least half of the firms will always choose the L-market. Since $\frac{\partial i^e}{\partial \gamma} = \frac{2 \cdot b - t \cdot l}{t \cdot l \cdot (\gamma - 1)} > 0$, then the coefficient γ can be considered as a coefficient of firms agglomeration. Equating $i^e = n$, we find the coefficient of asymmetry at which there will be a full agglomeration of firms on the L-market: $\gamma \ge \frac{2 \cdot b + t \cdot l \cdot (n-2)}{2 \cdot b - n \cdot t \cdot l}$.

We can summarize results in

Theorem 1. In the barbell model, under Cournot competition, increasing of firms numbers, markets sizes asymmetry and potential of markets will be conduce to agglomeration of firms on the large market. The increase in unit transportation costs will be conduce to dispersion of firms.

We note that the result obtained coincides with the effect of transport costs on agglomeration processes in models of the new economic geography **[11]**.

Let us analyze a profits of other firms, depending on the choice of the equilibrium firm. If the equilibrium firm is located on the L-market then a profits of the other firms are equal.

$$F_{i,i<i^e}^e(x_{i^e} = 0) = \frac{\gamma \cdot (b + t \cdot l \cdot (n - i^e))^2}{k \cdot (n+1)^2} + \frac{(b - t \cdot l \cdot (n - i^e + 1))^2}{k \cdot (n+1)^2}, \qquad (8)$$

$$F_{i,i>i^e}^e(x_{i^e} = 0) = \frac{\gamma \cdot (b - t \cdot l \cdot (i^e + 1))^2}{k \cdot (n+1)^2} + \frac{(b + t \cdot l \cdot i^e)^2}{k \cdot (n+1)^2}.$$

If the equilibrium firm is located on the S-market then a profits of the other firms are equal

$$F_{i,i<i^e}^e(x_{i^e} = l) = \frac{\gamma \cdot (b + t \cdot l \cdot (n - i^e + 1))^2}{k \cdot (n+1)^2} + \frac{(b - t \cdot l \cdot (n - i^e + 2))^2}{k \cdot (n+1)^2}, \qquad (9)$$

$$F_{i,i>i^e}^e(x_{i^e} = l) = \frac{\gamma \cdot (b - t \cdot l \cdot i^e)^2}{k \cdot (n+1)^2} + \frac{(b + t \cdot l \cdot (i^e - 1))^2}{k \cdot (n+1)^2}.$$

Comparing **(8)** and **(9)**, we obtain

$$F_{i,i<i^e}^e(x_{i^e} = 0) = F_{i,i>i^e}^e(x_{i^e} = l),$$

$$F_{i,i>i^e}^e(x_{i^e} = 0) = F_{i,i<i^e}^e(x_{i^e} = l),$$

$$F_{i,i<i^e}^e(x_{i^e} = l) - F_{i,i>i^e}^e(x_{i^e} = l) = F_{i,i>i^e}^e(x_{i^e} = 0) - F_{i,i<i^e}^e(x_{i^e} = 0) =$$

$$= F_{i,i<i^e}^e(x_{i^e} = l) - F_{i,i<i^e}^e(x_{i^e} = 0) = F_{i,i>i^e}^e(x_{i^e} = 0) - F_{i,i>i^e}^e(x_{i^e} = l) =$$

$$= \frac{t^2 \cdot l^2 \cdot (\gamma + 1)}{k \cdot (n+1)}.$$

Thus, the presence of the equilibrium firm in the market reduces profits of its neighboring firms and increases profits of firms in another market. The difference in the profits of firms in different markets does not depend on the location of the equilibrium firm.

Now we consider some special cases of the model.

Let us assume that firms compete in conditions of the full symmetry. In this case, all firms are located in one of the markets, a size of the markets are same, $x_1 = x_2 = ... = x_n$, $\gamma = 1$. In a future, the market, on which the i-th firm is located, we will call as "home market".

From **(3)** and **(4)** we find the equilibrium volumes of supplies and profits of the i-th company:

$$\left(q_i^L\right)^{sym} = \frac{b - t \cdot x_i}{k \cdot (n+1)},\ \left(q_i^S\right)^{sym} = \frac{b - t \cdot (l - x_i)}{k \cdot (n+1)},$$
$$F_i^{sym} = \frac{(b - t \cdot x_i)^2}{k \cdot (n+1)^2} + \frac{(b - t \cdot (l - x_i))^2}{k \cdot (n+1)^2}. \qquad (10)$$

From **(10)** we received that a volume of supplies and profit of the i-th firm on the home market is always higher. In conditions of full symmetry, firms have no preferences in choosing a market, since a total volume of supplies and profits are equal:

$$\left(q_i^L\right)^{sym}(x_i = 0) + \left(q_i^S\right)^{sym}(x_i = 0) =$$
$$= \left(q_i^L\right)^{sym}(x_i = l) + \left(q_i^S\right)^{sym}(x_i = l) = \frac{2 \cdot b - t \cdot l}{k \cdot (n+1)}, \qquad (11)$$
$$F_i^{sym}(0) = F_i^{sym}(l) = \frac{b^2 + (b - t \cdot l)^2}{k \cdot (n+1)^2}.$$

To select a market, firms need additional conditions. We note that the worst spatial decision of firms is the central agglomeration: $\left(x_i^{worst}\right)^{sym} = l/2$.

Now consider the case when firms compete in conditions of markets symmetry and location asymmetry, i.e. $\gamma = 1$, $x_1 \le x_2 \le ... \le x_n$.

From **(7)** we find that for the same size of markets, the index of the equilibrium firm is: $i^e(\gamma = 1) = (n+1)/2$. Thus, in the equilibrium state, firms will be distributed roughly equally between markets. Assume that the equilibrium number of firms in each market is equal $n/2$.

From **(3)** and **(4)** we find the equilibrium volumes of supplies and profits of the i-th company:

$$\left(q_i^L\right)^{loc\_asym} = \frac{b - t \cdot (n+1) \cdot x_i + t \cdot l \cdot n/2}{k \cdot (n+1)},$$
$$\left(q_i^S\right)^{loc\_asym} = \frac{b - t \cdot l + t \cdot (n+1) \cdot x_i - t \cdot l \cdot n/2}{k \cdot (n+1)}, \qquad (12)$$
$$F_i^{loc\_asym} = \frac{(b - t \cdot (n+1) \cdot x_i + t \cdot l \cdot n/2)^2}{k \cdot (n+1)^2} +$$
$$+ \frac{(b - t \cdot l + t \cdot (n+1) \cdot x_i - t \cdot l \cdot n/2)^2}{k \cdot (n+1)^2}.$$

The profit and volume of supplies of firms in both markets are the same:

$$F_i^{loc\_asym}(0) = F_i^{loc\_asym}(l) =$$
$$= \frac{(b + t \cdot l \cdot n/2)^2 + (b - t \cdot l \cdot (n/2 + 1))^2}{k \cdot (n+1)^2},$$
$$\left(q_i^L\right)^{loc\_asym}(x_i = 0) + \left(q_i^S\right)^{loc\_asym}(x_i = 0) = \qquad (13)$$
$$= \left(q_i^L\right)^{loc\_asym}(x_i = l) + \left(q_i^S\right)^{loc\_asym}(x_i = l) = \frac{2 \cdot b - t \cdot l}{k \cdot (n+1)}.$$

The worst spatial decision of firms is the central agglomeration: $\left(x_i^{worst}\right)^{loc\_asym} = l/2$.

To analyze the effects of the location asymmetry, let us compare **(10)** and **(12)**. Change of the deliveries:

$$\left[\left(q_i^L\right)^{sym} + \left(q_i^S\right)^{sym}\right] = \left[\left(q_i^L\right)^{loc\_asym} + \left(q_i^S\right)^{loc\_asym}\right],$$
$$\left(q_i^L\right)^{loc\_asym} - \left(q_i^L\right)^{sym} = \left(q_i^S\right)^{sym} - \left(q_i^S\right)^{loc\_asym} =$$
$$= \frac{t \cdot l \cdot n/2 - t \cdot n \cdot x_i}{k \cdot (n+1)}.$$

Change of the profit:

$$F_i^{loc\_asym} - F_i^{sym} = \frac{n \cdot t^2 \cdot l^2 \cdot (n+2)}{2 \cdot k \cdot (n+1)^2} > 0.$$

Thus, as a result of an appearance of the location asymmetry, the total volume of supplies did not change, the supplies were redistributed in favor of the home market, the profits of all firms increased.

Conclusions and prospects for further research. In this paper we generalize the model **[1]** to the case of a set of firms. This allowed us to find an equilibrium distribution of firms between markets. It is obtained that increasing of firms numbers, markets size asymmetry and potential of markets will be conduce to agglomeration of firms in the large market. The increase in unit transportation costs will be conduce to dispersion of firms. It is shown that the presence of the equilibrium firm in the market reduces profits of its neighboring firms and increases profits of firms in another market. The difference in the profits of firms in different markets does not depend on the location of the equilibrium firm. It is received, that from the point of view of profit the worst for firms is the full symmetry.

In the future supposed simulation of equilibrium in the barbell model under impact of other asymmetries.

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